Lesson 13: Equations

Lesson Overview

In this lesson, you will explore the equations used throughout this course. Each equation is presented with an example problem and the solution.

Upon completion of this lesson, you will be able to:

- Identify and understand the different formulas used throughout the course.
- Accurately apply the correct formula to real-life situations.

This lesson should take approximately 1 hour to complete.

Generalized Gaussian Equation for Ground Level Release Downwind Centerline Value

This equation is used to calculate the concentration of radioactive material in the air directly downwind from a release that occurs at ground level.

Chi represents the concentration of curie per meter cubed. This equation will be used to solve for chi. Note: curie per meter cubed is equivalent to micro-curie per centimeter cubed; chi can be expressed in either format.

This equation's purpose is to determine the concentration of radioactive material at different distances downwind from a ground-level release. The variable "x" represents the distance from the release, in meters, being measured.

Since the equation is measuring the concentration directly on the centerline, "y" is 0 in this equation. Similarly, this equation measures the concentration at ground level, so "z," which is the meters above the surface being measured, is also 0.

The height of release for this equation is also 0, since it is a ground level release.

As you can observe, "x" doesn't appear in the equation itself. Rather, it will be used to find sigma-sub-y and sigma-sub-z. Those values will be found using graphs which we'll explore soon.

The other values needed are "Q" and mu. "Q" is the release rate, measured in curies per second.

Mu represents wind speed at ground level, which is measured in meters per second.

Sigma-sub-y and sigma-sub-z are both values measured in meters. Determining their values requires the use of established resources.

This table and the following figures come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Sigma-sub-z can be found using Figure 13-3. The value of sigma-sub-z is determined in the same manner as the value of sigma-sub-y.

Now back to the equation, all of the values necessary to find chi are available. For example: Given a release rate of 3 curies per second, on an overcast night, with a wind speed of 7 meters per second, what is the centerline concentration 3 kilometers from the release point?

Using Table 13-1, the neutral class D should be applied due to overcast conditions, which allows you to use Figure 13-2 to find sigma-sub-y: 190 meters.

Next use Figure 13-3 to find sigma-sub-z: 65 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The centerline concentration 3 kilometers downwind of the ground level release is 1.1 times 10 to the negative 5th curie per meter-cubed.

Generalized Gaussian Equation for Elevated Release Downwind Centerline Value

This equation is used to calculate the concentration of radioactive material in the air directly downwind from a release that occurs at an elevated release.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

This equation's purpose is to determine the concentration of radioactive material at different distances as it is released at different heights. The variable "x" represents the distance from the release, in meters.

This equation solves for chi at 0 meters from centerline and 0 meters above the surface.

The variable "H" represents the effective height of the release, in meters.

As you can observe, "x" doesn't appear in the equation itself. Rather, it will be used to find sigma-sub-y and sigma-sub-z. Those values will be found using graphs which we'll explore soon.

The other values needed are "Q" and mu. "Q" is the release rate, measured in curies per second.

Mu represents wind speed at the release height, which is measured in meters per second.

Sigma-sub-y and sigma-sub-z are both values measured in meters. Determining their values requires the use of established resources.

This table and the following figures come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Sigma-sub-z can be found using figure 13-3. The value of sigma-sub-z is determined in the same manner as the value of sigma-sub-y.

Now back to the equation, all of the values necessary to find chi are available. For example:

Given a release rate of 80 curies per second on an overcast morning, with a wind speed of 6 meters per second, and an effective release height of 60 meters, what is the centerline concentration 500 meters from the release point?

Using Table 13-1, the neutral class D should be applied due to overcast conditions, which allows you to use Figure 13-2 to find sigma-sub-y: 36 meters.

Next use figure 13-3 to find sigma-sub-z: 18.5 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The centerline concentration 500 meters downwind of an effective release height of 60 meters is 3.3 times 10 to the negative 5th curie per meter-cubed.

Generalized Gaussian Equation for Ground Level Release Off Centerline Value

This equation is used to calculate the concentration of radioactive material in the air at a location not directly downwind centerline after a ground level release.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

This equation's purpose is to determine the concentration of radioactive material at different distances not directly downwind centerline from a ground level release. The variable "x" represents the distance from the release, in meters.

"y" is the distance from centerline, in meters.

"z," which is the meters above the surface being measured, is 0.

The height of release for this equation is also 0, since it is a ground level release.

As you can observe, "x" doesn't appear in the equation itself. Rather, it will be used to find sigma-sub-y and sigma-sub-z. Those values will be found using graphs which we'll explore soon.

The other values needed are "Q" and mu. "Q" is the release rate, measured in curies per second.

Mu represents wind speed at the release height, which is measured in meters per second.

Sigma-sub-y and sigma-sub-z are both values measured in meters. Determining their values requires the use of established resources.

This table and the following figures come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Sigma-sub-z can be found using figure 13-3. The value of sigma-sub-z is determined in the same manner as the value of sigma-sub-y.

Now back to the equation, all of the values necessary to find chi are available. For example:

Given a release rate of 3 curies per second, on an overcast night, with a wind speed of 7 meters per second, what is the concentration 3 kilometers from the release point 500 meters off-center?

Using Table 13-1, the neutral class D should be applied due to overcast conditions.

which allows you to use Figure 13-2 to find sigma-sub-y: 190 meters.

Next use figure 13-3 to find sigma-sub-z: 65 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The concentration 3 kilometers from a release point, 500 meters off-center is 3.4 times 10 to the negative 7th curie per meter-cubed.

Calculating Off-Centerline Distance to a Desired Value from Centerline Value at a Specific Distance

A centerline concentration may have been determined by an air sample taken at a point downwind. From that concentration, an off-centerline concentration may be calculated.

“y” represents the distance from centerline, in meters. This equation will be used to find “y.”

Note that “y” is the distance *from* the centerline. The plume *width* is 2 times y.

Two different values of chi, as well as sigma-sub-y, are needed to solve for “y.”

The chi in the numerator represents the centerline concentration that has been taken at a point downwind.

The chi in the denominator is the concentration off-centerline.

Sigma-sub-y is a value measured in meters. Determining its value requires the use of established resources.

This table and the following figure come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Back to the equation, all of the values necessary to find "y" are available. For example:

Given a downwind centerline concentration at 3 kilometers of 1.1 times 10 to the negative 5th curie per meter cubed, on an overcast morning, at what distance from centerline is the concentration of 3.48 times 10 to the negative 7th curie per meter cubed?

Using Table 13-1, the neutral class D should be applied due to overcast conditions.

which allows you to use Figure 13-2 to find sigma-sub-y: 190 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The distance from the centerline is 500 meters.

Remember that "y" is the distance from the centerline in either direction, which means the plume of the concentration chosen is 1,000 meters wide.

Generalized Gaussian Equation for Elevated Release Off Centerline Value

This equation is used to calculate the concentration of radioactive material in the air at a location not directly downwind centerline after an elevated release.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

This equation's purpose is to determine the concentration of radioactive material at different distances as it is released at different heights. The variable "x" represents the distance from the release, in meters. "y" is the distance from centerline, in meters.

“z,” which is meters above the surface being measured, is 0.

The variable “H” represents the effective height of the release, in meters.

As you can observe, “x” doesn’t appear in the equation itself. Rather, it will be used to find sigma-sub-y and sigma-sub-z. Those values will be found using graphs which we’ll explore soon.

The other values needed are “Q” and mu. “Q” is the release rate, measured in curies per second.

Mu represents wind speed at the release height, which is measured in meters per second.

Sigma-sub-y and sigma-sub-z are both values measured in meters. Determining their values requires the use of established resources.

This table and the following figures come from the EPA’s *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed’s stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Sigma-sub-z can be found using figure 13-3. The value of sigma-sub-z is determined in the same manner as the value of sigma-sub-y.

Now back to the equation, all of the values necessary to find chi are available. For example:

Given a release rate of 80 curies per second on an overcast morning, with a wind speed of 6 meters per second, and an effective release height of 60 meters, at 500 meters downwind, what is the concentration 50 meters off-centerline?

Using Table 13-1, the neutral class D should be applied due to overcast conditions, which allows you to use Figure 13-2 to find sigma-sub-y: 36 meters.

Next use figure 13-3 to find sigma-sub-z: 18.5 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The concentration 50 meters off-centerline 500 meters downwind of an effective release height of 60 meters is 1.3 times 10 to the negative 5th curie per meter-cubed.

Fumigation (Atmospheric Mixing Lid)

The mixing lid acts as a ceiling, keeping radioactive material concentrated near the surface. This equation can be used to determine the concentration of radioactive material in this instance.

Chi represents the concentration of curie per meter cubed, or microcurie per centimeter cubed. This equation will be used to solve for chi.

The variable "x" represents the distance from the release, in meters.

This equation solves for chi at 0 meters from centerline, 0 meters above the surface, and a 0-meter effective height of release.

As you can observe, "x" doesn't appear in the equation itself. Rather, it will be used to find sigma-sub-y. That value will be found using figures which we'll explore soon.

The other values needed to find chi are "Q," "L," and mu.

"Q" is the release rate, measured in curies per second.

"L" is the height of the lid, in meters.

Mu represents wind speed at effective release height, which is measured in meters per second.

Sigma-sub-y is measured in meters. Determining its value requires the use of established resources.

This table and the following figures come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

With that information, the concentration under a lid can be determined. For example:

Given a release rate of 10 curies per second, on an overcast night, a wind speed of 7 meters per second, and the lid at 500 meters, what is the centerline concentration at 5 kilometers?

Using Table 13-1, the neutral class D should be applied due to overcast conditions, which allows you to use Figure 13-2 to find sigma-sub-y: 300 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The centerline concentration at 5 kilometers is 3.8 times 10 to the negative 6^{th} curie per meter cubed.

Building Wake

Large structures at the release point alter the dispersion of radioactive material. The effect washes out at 1 to 2 miles, or 2 to 3 kilometers. This equation is used to calculate the concentration of radioactive material in the air directly downwind from a release that occurs at ground level with a building altering the dispersion.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

This equation's purpose is to determine the concentration of radioactive material at different distances as it is affected by the wake of large structures. The variable "x" represents the distance from the release, in meters.

This equation solves for chi at 0 meters from centerline, 0 meters above the surface, and a 0-meter effective height of release.

As you can observe, "x" doesn't appear in the equation itself. Rather, it will be used to find sigma-sub-y and sigma-sub-z. Those values will be found using graphs which we'll explore soon.

The other values needed are "Q," mu, "c" and "A." "Q" is the release rate, measured in curies per second.

Mu represents wind speed at ground level, which is measured in meters per second.

"c" is the shape factor, which is usually zero point five (typical building shape).

"A" is the cross-sectional area in meters squared. This is equal to the building's height multiplied by width.

Sigma-sub-y and sigma-sub-z are both values measured in meters. Determining their values requires the use of established resources.

This table and the following figures come from the EPA's *Workbook of Atmospheric Dispersion Estimates*, published in 1970. To make a more accurate dose projection, use this table to estimate the combined atmosphere and wind speed's stability class.

Once the stability class is determined, sigma-sub-y can be found using this figure. The log-log graph is read along the bottom out to a desired distance, up to the appropriate stability class line, then left to the sigma-sub-y value, in meters.

For example, at 5 kilometers downwind and a D stability, the sigma-sub-y is 300 meters.

Sigma-sub-z can be found using figure 13-3. The value of sigma-sub-z is determined in the same manner as the value of sigma-sub-y.

Now back to the equation, all of the values necessary to find chi are available. For example: given a release rate of 10 curies per second, on an overcast day, with a wind speed of 7 meters per second, and a containment building 75 meters tall and 50 meters wide, which yields a cross-sectional area of 3750 meters squared, what is the concentration at 1 kilometer?

Using Table 13-1, the neutral class D should be applied due to overcast conditions, which allows you to use Figure 13-2 to find sigma-sub-y: 70 meters.

Next use Figure 13-3 to find sigma-sub-z: 32 meters.

Now back to the equation, all the values need to be substituted in.

Then it can be solved.

The concentration at 1 kilometer is 1.6 times 10 to the negative 4th curie per meter cubed.

Volumetric Diffusion

Radioactive material will remain concentrated while traveling up or down a valley unless there is a strong crosswind. This equation can be used to determine the concentration of radioactive material as it moves along a valley.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

The variable “x” represents the distance from the release, in meters.

This equation solves for chi at 0 meters from centerline, 0 meters above the surface, and a 0-meter effective height of release.

The values needed to find chi are “Q,” mu, “H” and “W.”

“Q” is the release rate, measured in curies per second.

Mu represents wind speed at the release height, which is measured in meters per second.

“H” is the cross-sectional height of the valley walls,

and “W” is the cross-sectional width of the valley.

With that information, the concentration in a valley can be determined. For example:

Given a release rate of 10 curies per second, a wind speed of 7 meters per second, a valley wall height of 500 meters, and a valley width of 500 meters.

What is the concentration at 5 kilometers?

All the values need to be substituted in.

Then it can be solved.

The concentration at 5 kilometers is 5.7 times 10 to the negative 6^{th} curie per meter cubed.

Dry Deposition

Deposition is the physical settling or placing of material onto a surface.

W-sub-d represents the surface concentration of curie per meter squared. This equation will be used to solve for W-sub-d.

This equation’s purpose is to determine the concentration of radioactive material a plume leaves on a surface. The variable chi represents the plume’s concentration over the surface, in curie per meter cubed.

V-sub-d represents deposition velocity, in meters per second. Deposition velocity varies depending on what is being measured.

EPA 400-R-92-001 deposition rates are as follows:

For Iodine, it is 1 centimeter per second, or 1 times 10 to the negative 2nd meters per second.

For particulates, it is 0.1 centimeter per second, or 1 times 10 to the negative 3rd meters per second.

"t" stands for the length of time the plume is above the given point at which you wish to calculate the deposition, in seconds.

All of the values necessary to solve for W-sub-d are available. For example:

Given an airborne concentration above the surface equal to 1.11 times 10 to the negative 5th curie per meter cubed, in a plume containing Iodine, over the area for 2 hours.

What is the surface concentration of iodine using the EPA deposition rate?

The deposition rate for Iodine is 1 times 10 to the negative 2nd meters per second, and hours must be converted to seconds.

All the values need to be substituted in.

Then it can be solved.

The surface concentration is 8 times 10 to the negative 4th curie per meter squared.

Estimating Concentrations Using Diffusion Graphs

An atmospheric dispersion coefficient log-log graph has been developed for each stability class: A–F.

When combined with this formula, they effectively replace the need to calculate concentrations using sigma-sub-y, sigma-sub-z, effective height, and fumigation graphs.

Chi represents the concentration of curie per meter cubed, or micro-curie per centimeter cubed. This equation will be used to solve for chi.

"Q" is the release rate, measured in curies per second.

Mu represents wind speed at the release height, which is measured in meters per second.

The value chi times mu over Q is found using the graphs, and is measured in meters to the negative 2^{nd} power.

To use the graphs, first select the graph for the appropriate stability class.

For example: given a ground level release in a class D stability rating, with a wind speed of 7 meters per second, and a release rate of 3 curies per second, what is the concentration at 3 kilometers?

Using the D stability graph, move out 3 kilometers across the x-axis, and up to the H=0 line, to return the value 2.8 times 10 to the negative 5^{th} meters to the negative 2^{nd}.

Now, back to the formula.

All the values need to be substituted in.

Then it can be solved.

Chi is 1.2 times 10 to the negative 5^{th} curie per meter cubed.

Estimating Distances for Concentrations Using Diffusion Redaction Factor Table

Once a concentration has been measured or calculated for a specific distance and stability class, distances at which specified concentrations, or doses, would be found may be estimated by using this chart.

Care must be used when estimating doses using this chart. The release must be at ground level and there cannot be an inversion lid.

First, find the reduction factor.

For example, given a calculated dose of 5 rem at 5 kilometers from the release point, and a class F stability, how far from release would someone have to be to receive 1 rem?

Substitute in the values and solve to find a reduction factor of 5.

Look down the chart to find the reduction factor you calculated and look across to the applicable stability class column and determine the applicable distance ratio: 3.5.

To find the distance for the desired dose, we multiply the distance ratio by the distance at which the dose was measured.

Someone would need to be 17.5 kilometers from the source to receive a dose of 1 rem.

Minimum Requirements for Detecting Concentrations

NUREG-0654 requires that counting systems be capable of measuring radioiodine concentrations in air as low as 1 times 10 to the negative 7th microcurie per cm-cubed.

First, the minimum detectable limit must be calculated. Minimum detectable limit is measured in counts-per-minute.

The method used is different depending on whether the system involved is a digital system or an analog system.

For digital, only one piece of data must be obtained in order to calculate the Minimum Detectable Limit.

B is equal to the background, in counts per minute.

For analog, there is an additional requirement to solve.

RC refers to the meter time constant, provided by the manufacturer.

The detector's actual concentration measurement capability can be calculated, now that the minimum detectable limit has been determined.

The concentration is measured in microcurie per centimeter cubed, and sample volume is measured in centimeter cubed.

The count yield for this equation is measured in counts-per-minute over micro-curie.

Collection efficiency is the percent of the sampled material (in this case iodine) retained in the cartridge.

Detector efficiency is counts per disintegration. However, 2.22 times 10 to the 6th disintegrations per minute equals one micro-curie. We must use this in our conversion.

The necessary conversion: assuming a typical detector efficiency of 0.25%, or .25 count-per-minute for every 100 disintegrations-per-minute, converting the detector's count to a count yield in counts-per-minute over micro-curie involves the following conversion formula. (0.25% is a typical efficiency for GM pancake probes on a silver zeolite cartridge. A 1" or 2" NaI probe may have efficiencies near 10%.) We now have a number equivalent to (CY)(CE) for our system.

Now back to the equation, all of the values necessary to find “C” are available. For example:

Given an air sample volume of 60 liters per minute for 5 minutes, and a background count of 600 counts per minute, with a detector efficiency of 0.25 percent, and a collection efficiency of 90 percent, find if this digital system is capable of detecting the required minimum radio iodine concentration.

First the MDL must be calculated, so use the equation for digital systems.

Substitute in the background count, and solve.

The minimum detectable limit is 49 counts per minute.

The count yield must be converted to counts-per-minute over micro-curie with collection efficiency included.

Using the conversion equation, the count yield times the collection efficiency is 5.00 times 10 to the 3rd (5,000) counts-per-minute per micro-curie

Now back to the equation, all the values need to be substituted in, then it can be solved.

The system has the capacity of detecting concentrations of 6.53 times 10 to the negative 8th micro-curie over centimeter cubed.

It does meet the minimum requirement. If the required detection limit had not been met, then the sample volume should be increased to lower the detection limit. Sample volume should not be increased by increasing the flow rate because collection efficiency depends on the flow rate. It should be increased by extending the sampling time.

Determining Airborne Concentration from Air Sample

This formula can be used to calculate the airborne concentration of radioactive material from an air sample.

The equation solves for “C,” which is concentration, measured in microcurie per centimeter cubed.

GC stands for gross count, measured in counts per minute, and BC stands for background count, also measured in counts per minute.

“CE” is the collector efficiency, expressed as a decimal. For a SilverZeolite cartridge, this is 90%.

“DE” is detector efficiency, also expressed as a decimal. For a GM Pancake probe with a SilverZeolight cartridge, this is 0.25%.

“ASV” represents air sample volume, which is found using a separate formula:

“t” represents sample time, in minutes.

“ASR” stands for air sampler rate, which you can find with another equation.

Air sampler rate is volume, in cubic feet, over sample time, in minutes.

Air sample volume is returned in centimeters cubed.

With that information, the airborne concentration can be determined. For example:

Given a gross count of 25,306 curies per minute,

a background count of 216 curies per minute,

an air sampler rate of 2 cubic feet per minute, for 5 minutes,

a collector efficiency of 0.9, or 90%

and a detector efficiency of 0.0025, or 0.25%

what is the airborne concentration of the sample?

The air sample volume must be found first:

Use the air sampler rate.

The air sample volume is 2.8 times 10 to the 5^{th} centimeters cubed.

Now back to the equation, all of the values necessary to find “C” are available.

All the values need to be substituted in.

Then it can be solved.

The airborne concentration of the sample is 1.8 times 10 to the negative 5^{th} microcurie per centimeter cubed.

Lesson Summary

Now that you've completed this lesson, you can:

- Calculate the concentration of radioactive material in situations including disparate factors:
  - Elevation of the release
  - Distance from centerline
  - Height of atmospheric mixing lid
  - Dispersion caused by large structures
  - Diffusion into a defined volume such as a valley
- Determine the minimum detectable limit for counting systems
- Utilize tools found in the EPA's *Workbook of Atmospheric Dispersion Estimates* (1970):
  - Atmospheric Stability Class table
  - Diffusion Graphs
  - Reduction Factor Table

Congratulations on completing the Radiological Accident Assessment Concepts Pre-Course!